EXPERIMENTAL DETERMINATION OF SKIN FRICTION COEFFICIENT IN A TURBULENT BOUNDARY LAYER WITH A LONGITUDINAL PRESSURE GRADIENT

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A study is made of the applicability of indirect methods for the determination of the local skin friction coefficient in a turbulent boundary layer with a longitudinal pressure gradient.

§1. The experimental determination of the local skin friction coefficient c_f for flow with a longitudinal pressure gradient around a body is one of the most complex, and as yet unsolved, problems in experimental aerodynamics. The method of weight measurement of frictional force by means of a "floating" element is the most reliable for gradient-free flow of a liquid or gas, since this method is not associated with any assumptions about the nature of the flow in the boundary layer. However, the method of weight measurement of frictional force is practically unusable when there is a longitudinal pressure gradient present in the flow ($dP/dx \neq 0$), since the measurement of frictional force is then associated with the development of an extremely complex system of corrections for the displacement of the "floating" element produced by the pressure forces. As a result, one of the main advantages of the experimental technique using a "floating" element is lost; this is the fact that no preliminary calibration is required for weight measurement of the frictional force except for a determination of the elastic constant of the weight system. This led to a situation where mainly those indirect methods for the determination of skin friction were being used in experimental practice when $dP/dx \neq 0$ which were developed for gradient-free flow of liquid or gas as the simplest and most practical in use.

Indirect methods for the determination of c_f are mainly based on the measurement of velocity (or of its distribution) in the boundary layer with the universality of the velocity distribution laws in the boundary layer being assumed to exist. At the present time, however, the necessary information is lacking which would make it possible to judge the degree of suitability of indirect methods for the determination of skin friction in a turbulent boundary layer with a longitudinal pressure gradient. This results primarily from the absence of reliable methods for calibrating the indirect methods under these conditions, which eliminates the possibility of an objective evaluation. Nevertheless, there is a basis for assuming that indirect methods successfully used for the determination of c_f in gradient-free flow can also be used in the case of flows with comparatively small longitudinal pressure gradients, since it is considered that the velocity distribution law in a boundary layer in the flow region where the effect of viscosity is directly felt (wall law) is only slightly sensitive to external conditions, i.e., to the influence of a longitudinal pressure gradient. However, the limits of applicability of the indirect methods have not been determined rigorously, which considerably limits their practical use.

In this situation, there is undoubted interest in making a comparative analysis under identical conditions of gradient flow of the efficiency of various indirect methods for the determination of skin friction with simultaneous checking of the assumptions made in the creation of these methods with respect to the nature of the flow in the boundary layer for the actual conditions under which measurement is made. It can be assumed that such an analysis makes it possible to obtain the information necessary to arrive at some judgment as to the limits of applicability of the various indirect methods.

This paper presents the results of a comparison of the eight indirect methods most common in experimental practice under conditions where the pressure gradient varies both smoothly and abruptly along the flow, also including cases where there is a strong effect of prehistory on the development of the turbulent boundary layer.

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§ 2. Indirect methods for measuring skin friction which are based on the use of the wall law

$$\frac{u}{u_*} = f\left(\frac{yu_*\rho}{\mu}\right),$$

which follows from the logical premise concerning the unambiguous dependence of the shear stress at the wall, $\tau_W = \rho u_*^2$, on the velocity u at a distance y from the wall, on the viscosity μ, and on the density ρ, can be divided into two groups. One of them includes measurement of velocity in the immediate neighborhood of the wall in the region of the viscous sublayer of the turbulent boundary layer; the other includes measurement of velocity outside the viscous sublayer in the transition region and in the turbulent core of the boundary layer. The methods in the first group are mainly based on the assumption of a linear velocity distribution in the viscous sublayer, for which the value of the shear stress can be found from the relation

$$\tau_w = \mu \frac{u}{y},$$

and the methods in the second group are based on the assumption of the universality of the logarithmic velocity distribution law.

For the measurement of velocity in the immediate neighborhood of a wall in the region of the viscous sublayer, one ordinarily uses either flat micropitot tubes or surface devices. In the first case, the shear stress is determined from the initial slope of the linear velocity distribution in the viscous sublayer measured in the immediate vicinity of the wall. When surface devices are used, the value of τ_W is determined from the value of the velocity averaged over the height of a surface device located on the wall. In both cases it is necessary to consider the effect of the nearness of the wall, of the transverse velocity gradient, and of flow viscosity on readings from the micropitot tube or surface device [1, 2]. For surface devices made from safety-razor blades and deeply submerged in the viscous sublayer, the following calibration relation is recommended [2]:

$$\frac{\tau_w h^2}{4\rho\nu^2} = 0.3255\left(\frac{\Delta P h^2}{4\rho\nu^2}\right)^{0.7335}, \tag{1}$$

where $\Delta P = P_0 - P_{st}$ is the measured pressure differential between total and static pressure, which determines the average value of the velocity u over the height of the device; y = h is the height of the surface device up to its sharp front edge.

Also in the first group of methods is the fence method [3], which consists of measurement of the difference between the pressures before and behind a fence that projects a few hundredths of a millimeter above the surface along which flow occurs, and the method for determination of c_f by means of a probe with a heated element mounted flush with the surface along which flow occurs [4].

Assuming linearity of the velocity distribution near the wall, the difference between the pressures before and behind the fence will be proportional to the frictional stress at the wall:

$$\Delta P = k_1 \tau_w, \tag{2}$$

where the coefficient k_1 depends on the height of the fence and is determined by calibration.

The probe with heated element (thermal probe) has received widespread acceptance in foreign practice for the experimental determination of surface friction. As has been shown [4], the dimensionless coefficient of heat transfer from the heated element of the probe to the gas flow in the immediate neighborhood of the wall is proportional to the cube root of the frictional stress on the wall:

$$\mathrm{Nu} = k_2 \tau_w^{1/3}, \tag{3}$$

where Nu = const $(I^2R/\Delta T)$. Here R and I are the resistance and the current flowing through the heated element; ΔT is the difference between the temperature of the heated element of the probe and the temperature of the oncoming flow.

The universal nature of the wall law was used by Clauser [5] for determination of the local skin friction coefficient from measurements of the velocity distribution in the turbulent core of the boundary layer and also by Preston [6] for measurements of the velocity with a circular pitot tube of diameter D located directly on the surface along which flow occurred. Using the logarithmic velocity distribution law

$$\frac{u}{u_*} = 5.3 \lg \frac{yu_*}{\nu} + 5.8, \tag{4}$$

where the coefficients were selected on the basis of best agreement with the experimental data for gradient-free flow [7], and the equalities

$$\frac{u}{u_*}=\frac{u}{u_\infty}\left(\frac{2}{c_f}\right)^{1/2};\quad \frac{u_* y}{\nu}=\frac{u_\infty y}{\nu}\left(\frac{c_f}{2}\right)^{1/2};\quad c_f=\frac{2\tau_w}{\rho u_\infty^2},$$

one can obtain the following relation for the determination of the local skin friction coefficient:

$$\frac{u}{u_\infty}=\left(\frac{c_f}{2}\right)^{1/2}\left\{5.3\lg\frac{u_\infty y}{\nu}+5.3\lg\left(\frac{c_f}{2}\right)^{1/2}+5.8\right\}. \tag{5}$$

By using the Preston method, the requirement for measurement of the velocity near the wall is considerably softened in comparison with the case where friction is determined from the initial slope of the experimental curve for the velocity distribution in the viscous sublayer or from surface devices deeply submerged within the viscous sublayer. When the size of a circular pitot tube located on the surface along which flow occurs is such that it includes the viscous sublayer and the transitional (buffer) region between the viscous sublayer and the turbulent core of the boundary layer, the following calibration relation is recommended [8]:

$$y^*=0.8287-0.1381x^*+0.1437x^{*2}-0.006x^{*3} \tag{6}$$

for $1.5 < y^* < 3.5$, where $y^* = \log(\tau_W D^2/4\rho\nu^2)$ and $x^* = \log(\Delta P D^2/4\rho\nu^2)$. If the tube includes the region of the turbulent core of the boundary layer but does not extend beyond the region where the wall law is satisfied, the calibration relation is written in the form† [9]

$$x^*=y^*+2\lg(1.95y^*+4.404) \tag{7}$$

where $3.5 < y^* < 5.3$.

Standing somewhat apart in the set of indirect methods is the method of Ludweig and Tillmann [10] which is based on measurement of the velocity distribution over the entire thickness of the boundary layer, including the external region of the turbulent boundary layer where viscosity is not a dominant factor. Based on their experiments in a boundary layer with a positive pressure gradient, Ludweig and Tillmann proposed the following empirical relation:

$$c_f=\frac{0.246}{10^{0.678H}\mathrm{Re}^{**0.268}}, \tag{8}$$

where

$$H=\frac{\delta^*}{\delta^{**}}=\frac{\int\limits_0^\infty\left(1-\frac{u}{u_\infty}\right)dy}{\int\limits_0^\infty\frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)dy};\quad \mathrm{Re}^{**}=\frac{u_\infty\,\delta^{**}}{\nu}.$$

A similar simple relation was also obtained by Head and Patel [11]:

$$c_f=\exp(aH+b), \tag{9}$$

where $a = 0.02-0.387c + 0.028c^2 - 0.0007c^3$; $b = 0.192 - 0.835c + 0.063c^2 - 0.002c^3$; $c = \ln \mathrm{Re}^{**}$. Equation (9) was obtained by theoretical methods from consideration of the integral relation of momenta written for the case $dP/dx \neq 0$ with the nonequilibrium nature of the flow taken into consideration. It should be pointed out here that Eqs. (8) and (9) are recommended as theoretical equations but they can also be used for experimental determination of c_f.

§3. The indirect methods described in Sec. 2 and used in our experiments for the determination of skin friction when $dP/dx \neq 0$ were first tested under the simplest conditions of gradient-free flow.

Experimental values of c_f obtained by the weight method by means of a "floating" element (see curve I, Fig. 1) were taken as control values for skin friction. Values of c_f determined from Eqs. (1) and (5)-(9) are compared with curve I in Fig. 1b. Also shown are values of c_f obtained from the initial slope of the velocity distribution in the viscous sublayer measured with a flat micropitot tube and corrected for the effects of

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†The numerical values of the coefficients in Eqs. (6) and (7) are determined by selection of the coefficients A, B, and C in the logarithmic velocity distribution law in the transition region of the turbulent boundary, $u/u_* = A\log(yu_*/\nu + C) + B$, and in the turbulent core, $u/u_* = A\log(yu_*/\nu) + B$.

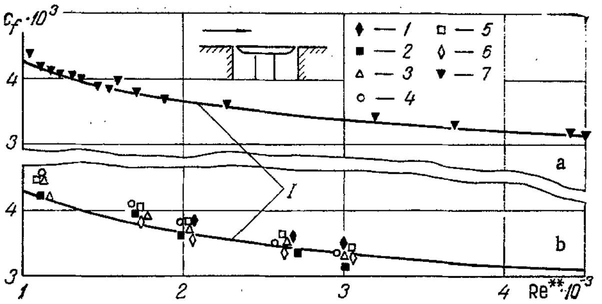

Fig. 1

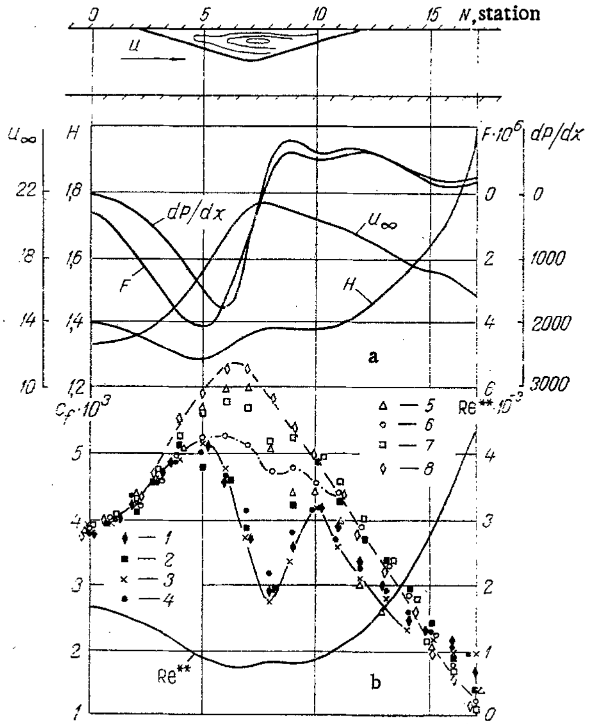

Fig. 2

Fig. 1. Dependence of the local skin friction coefficient measured by the weight method on Reynolds number for dP/dx = 0 (a); comparison of values of the local skin friction coefficient measured by indirect methods for dP/dx = 0 and various Reynolds numbers with values of c_f obtained by the weight method (b): 1) method of calibrated surface devices; 2) Ludweig–Tillmann method; 3) Head–Patel method; 4) method of initial slope of velocity profile in viscous sublayer; 5) Preston method; 6) Clauser method; 7) weight measurements.

Fig. 2. Variation of flow characteristics in the boundary layer (a) (u_∞, m/sec; dP/dx, N/m^3); comparison of values of local skin friction coefficient measured by various indirect methods for dP/dx $\neq$ 0 (b): 1) method of surface devices; 2) method of initial slope of velocity profile in viscous sublayer; 3) method of probe with heated element; 4) fence method; 5) Preston method; 6) Ludweig–Tillmann method; 7) Head–Patel method; 8) Clauser method.

viscosity, nearness to the wall, and velocity gradient on the readings of the micropitot tube [1]. Weight measurements of frictional force were also used to obtain calibration relations for the fence sensor and the probe with heated element, which require individual calibration [see Eqs. (2) and (3)]. In the weight determination of c_f, highly sensitive magnetoelectric weights [12] were used with a frictional force measurement range from 1 to 10 mg and inductive weights [13] with a measurement range from 5 to 50 mg. The "floating" element was prepared in accordance with the recommendations of [14] and was of rectangular shape with dimensions 15 × 30 mm and 4 × 12.5 mm, respectively.

As is clear from Fig. 1b, all experimental points are grouped around curve I independently of the method of measurement with a standard deviation of 4.5%. This indicates the excellent efficiency of the methods we selected for the determination of c_f under conditions of gradient-free flow.

A longitudinal pressure gradient in the boundary layer was created by means of a shaped insert installed in the operating portion of the wind tunnel. Figure 2a shows the variation in the values of dP/dx and in the dimensionless shape parameter $F = (\nu/\rho u_\infty^3)(dP/dx)$, which characterizes the longitudinal pressure gradient along the insert. Also shown is the variation of the velocity at the outer limit of the boundary layer and the variation of the shape parameter H of the boundary layer.

Values of local skin friction coefficients c_f determined at 18 measuring stations along the insert by means of indirect methods are shown in Fig. 2b. As is clear, up to $F = -4 \cdot 10^{-6}$ (fourth station) where a smooth and monotonic variation of the negative pressure gradient still occurs, all experimental points can be described independently of the method of measurement by a single common curve. Farther downstream, where d^2P/dx^2

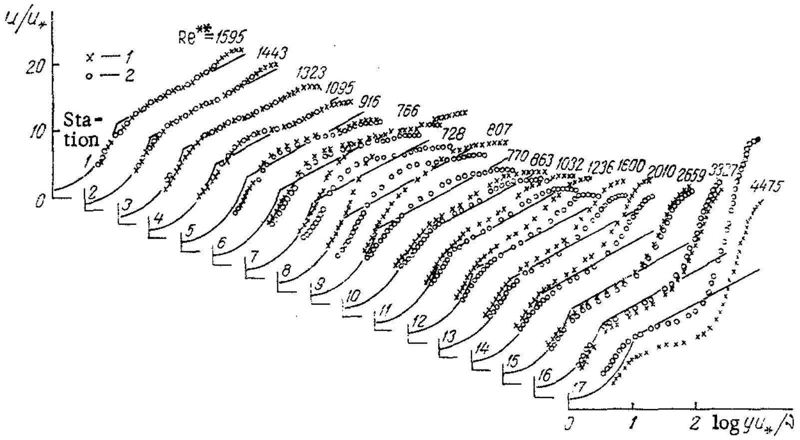

Fig. 3. Velocity distribution in a boundary layer with a longitudinal pressure gradient: 1) u_* from probe with heated element; 2) u_* from the Ludweig–Tillmann method.

changes sign and the subsequent drop in the value of the shape parameter begins to slow down (after which the parameter F begins to rise), splitting of the experimental points along two paths is observed. One of these paths includes experimental points obtained by means of methods based on the measurement of flow parameters in the immediate neighborhood of the wall within the viscous sublayer, and the other path includes experimental points obtained by means of methods based on measurement of velocity within the confines of the region, which is covered by the effect of the wall law or based on the velocity distribution over the entire thickness of the boundary layer. The spread of the experimental points decreases markedly starting at station 10 with dP/dx and the shape parameter F taking on positive values.

In Fig. 2b, the solid line connects experimental points obtained by a probe with a heated element,† and the dashed line connects experimental points obtained by means of Eq. (5) in the Clauser method. We consider these methods of measurement as most typical of the group of indirect methods specified above. Values of c_f obtained by the use of Eq. (8) from Ludweig and Tillmann are connected by the dashed-dot line occupying an intermediate position in Fig. 2b, which possibly results from the specific characteristics for which this relation was obtained. Indeed, although the empirical relation (8), which connects friction with the shape parameter H for the velocity profile in the boundary layer, was also obtained under an assumption of the validity of the wall law for a positive pressure gradient, the actual value of the friction in this case was taken by Ludweig and Tillmann from measurements made with a probe with a heated element located directly on the surface along which flow occurred.

We now compare the experimental distribution obtained for the values of c_f along the insert with the profiles of averaged velocity in the boundary layer measured at the same measuring stations.

In Fig. 3, the experimental velocity distribution in the boundary layer is compared with the logarithmic law (4), where $u_* = \sqrt{\tau_w/\rho}$ was determined both with a probe with heated element (points 1) and by means of Eq. (8) from Ludweig and Tillmann (points 2), i.e., by use of methods in the first and second groups.

As is apparent, up to station 4 where the experimental values of c_f are still unsplit and fall on a single common curve, a linear velocity distribution occurs in the viscous sublayer and the logarithmic velocity distribution law remains valid in the turbulent core of the boundary layer. Thus, the assumptions made as the basis of indirect methods belonging both to the first and second groups find experimental verification.

At stations 5-10, the logarithmic law (4) is not satisfied and a sharp deviation of experimental points from the logarithmic velocity distribution law is observed over the entire thickness of the boundary layer, while the velocity distribution in the viscous sublayer can still be assumed to be practically linear (see points 1 in Fig. 3). With a positive pressure gradient (stations 10-16), the wall law is satisfied somewhat better and this leads to convergence of the experimental values of c_f obtained by the different methods.

Consequently, in those cases where a marked change in the longitudinal pressure gradient occurs, where the effect of the prehistory of the boundary layer on its development is noticeable in the boundary layer, and where a tendency toward lamination of the averaged velocity profile is observed (see Fig. 3, stations 6-9), the

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†The contribution to the average friction from a pulsating component was insignificant in these experiments.

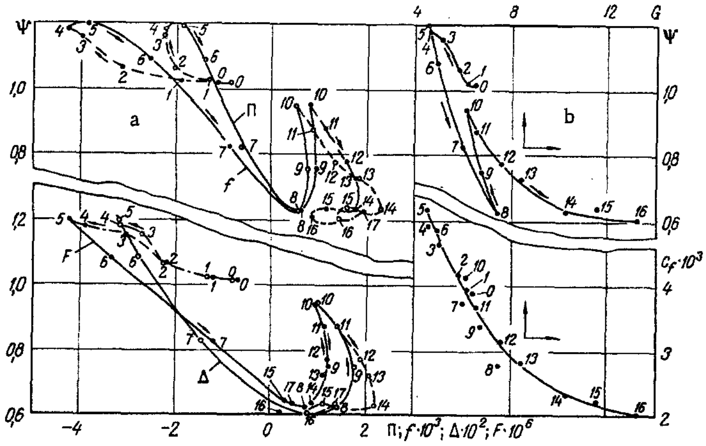

Fig. 4. Dependence of relative skin friction coefficient Ψ on the shape parameters for a longitudinal pressure gradient F, f, Δ, and Π(a); dependence of relative skin friction coefficient Ψ and skin friction coefficient c_f on the shape parameter G (b).

use of indirect methods based on the universality of the logarithmic velocity distribution law is not justified. At the same time, lamination of the turbulent boundary layer does not eliminate the possibility of using methods in the first group for the determination of c_f. The latter is in agreement with the conclusions of [15] where it was shown that the calibration equation for a probe with a heated element inserted flush with the surface along which flow occurs remains the same both in a turbulent and laminar boundary layer. This results from the fact that the thermal probe is deeply buried in the viscous sublayer where the flow is nearly laminar in any case.

Thus, the results presented provide a basis for supposing that in the determination of the skin friction coefficient in a turbulent boundary layer with a longitudinal pressure gradient preference should be given to those indirect methods based on a linear velocity distribution in the immediate neighborhood of the wall.

An attempt was made to represent the experimental values of c_f as a function of the shape parameter characterizing the longitudinal pressure gradient. As such a shape parameter, we tested the various forms

$$F = \frac{\nu}{\rho u_\infty^3}\frac{dP}{dx}; \quad f = \frac{\delta^{**}}{\rho u_\infty^2}\frac{dP}{dx}; \quad \Delta = \frac{\nu}{\rho u_*^3}\frac{dP}{dx} \text{ and } \Pi = \frac{\delta^*}{\tau_w}\frac{dP}{dx},$$

which are most widely used in experimental practice and which reflect various points of view about the structure of the shape parameter for a longitudinal pressure gradient. Figure 4 shows the dependence of the relative skin friction coefficient $\Psi = (c_f/c_{f_0})_{Re^{**}=\mathrm{idem}}$ on F, f, Δ, and Π. In the determination of Ψ, values of c_f in gradient flow were taken from experimental data obtained with a probe with a heated element and the values of c_{f_0} for the same Re** numbers were taken from experimental data obtained by the weight method in gradient-free flow (Fig. 1a). The numbers 0, 1, 2, . . . , 17 in Fig. 4 denote the number of the measuring station, and the arrows indicate the direction of passage.

As is clear from Fig. 4, the shape parameters F, f, Δ, and Π can be used for a description of the experimental values of Ψ only up to the fourth or fifth measuring station where there still is a smooth and monotonic variation of the negative pressure gradient. Farther downstream the dependence of Ψ on these shape parameters becomes nonunique with the nature of these relations being approximately the same and distinguished mainly by the degree of development of this nonuniqueness. The solid lines in Fig. 4a connect those experimental points which were obtained for an approximately constant Re** number (stations 5-10, Fig. 2b), which made possible the elimination of a possible effect of the Re** number on the value of the relative skin friction coefficient Ψ.

The nonuniqueness of the dependence of Ψ on the shape parameter for the pressure gradient is a result of the fact that the behavior of a turbulent boundary layer depends not only on local conditions (such as the local value of the pressure gradient, frictional shear stress on the wall, surface conditions, etc.), but also on prehistory in the development of the boundary layer. Since the prehistory in each specific situation can be different, a description of boundary-layer characteristics by only local parameters is not unique.

In Fig. 4b, values of the skin friction coefficient are presented as a function of the Clauser integral shape parameter [5]:

$$G=\int\limits_0^1\left(\frac{u_\infty-u}{u_*}\right)^2 d\,(y/\delta)\Bigg/\int\limits_0^1\left(\frac{u_\infty-u}{u_*}\right)d\,(y/\delta)=\left(\frac{2}{c_f}\right)^{1/2}\left(1-\frac{1}{H}\right),$$

which is related to the shape of the velocity profile in the boundary layer. As is evident, the nonuniqueness develops less here than in Fig. 4a; it also appears worthwhile to construct the relation $c_f(G)$ instead of $\Psi(G)$.

LITERATURE CITED

1. E. U. Repik, Inzh.-Fiz. Zh., 22, No. 3 (1972).
2. E. U. Repik, Tekh. Otchety Tsentr. Aero-Gidrodinam. Inst. im. N. E. Zhukovskii, No. 150 (1959).
3. N. I. Konstantinov and G. L. Dragnysh, Tr. Leningr. Politekh. Inst., No. 176 (1955).
4. D. A. Spence and G. L. Brown, J. Fluid Mech., 33, Part 4 (1968).
5. F. H. Clauser, J. Aeronaut. Sci., 21, No. 2 (1954).
6. D. Preston, in: Mekhanika [Period Collection of Translations of Foreign Articles], No. 6 (1955).
7. E. U. Repik, Tr. Tsentr. Aero-Gidrodinam. Inst., No. 1218 (1970).
8. V. Patel, J. Fluid Mech., 23, Part 1 (1965).
9. E. U. Repik and V. N. Tarasova, Tr. Tsentr. Aero-Gidrodinam. Inst. im. N. E. Zhukovskii, No. 1218 (1970).
10. H. Ludweig and W. Tillmann, TM 1285, NACA (1950).
11. M. R. Head and V. C. Patel, R and M 3643, ARC (1969).
12. V. I. Zimenkov, Inventor's Certificate No. 149919, Byul. Izobret., No. 17 (1962).
13. A. G. Taryshkin and V. V. Svirin, Author's Certificate 268680 (1969); Otk. Izobret., Prom. Obraz., Tovar. Znaki, No. 14 (1970).
14. S. Dhawen, TN 2567, NACA (1952).
15. B. Narayanan and V. Ramjee, J. Aeronaut. Soc. India, 20, No. 1 (1968).

DETERMINATION OF THE DISTANCE TO A TURBULENT CONE DURING THE INTERACTION BETWEEN LAMINAR AXISYMMETRIC FREE JETS

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Results are presented of an experimental investigation to determine the distance to the turbulent cone originating in a laminar feeding jet during its interaction with a laminar control jet.

Turbulent amplifiers [1, 2], whose operating principle is based on the forced turbulization of the axisymmetric free feeding submerged jet by a laminar control jet of the same type and on the deflection of the feeding jet, have recently been used extensively in jet pneumoautomation. In this connection, the question of determining the distance to the turbulent cone of the laminar feeding jet, which is needed for the computation of the static and dynamic characteristics of a turbulent amplifier, is urgent.

It is known that if mechanical obstacles, control jets, sound fields, etc., do not act on an axisymmetric laminar free jet, then the laminar jet becomes unstable for $Re_0 > 30$ [1], computed according to the stream in the capillary shaping the jet, and there is a transition to turbulence. The loss of the laminar jet stability is explained by vortex origination during its emergence from the capillary [3], which spoils the laminar structure by acting on the jet.

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